Percentages
Loans and Saving

Percentages

- Percent is the number per 100
- To convert a decimal to a percent, move the decimal point two digits to the *right*
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Examples % to Decimal

3.7% → 0.037

125.8% → 1.258

19% → 0.19

1.2% → 0.012

Examples: Decimal to %

$0.48 \rightarrow 48\%$

$0.031 \rightarrow 3.1\%$

$2.173 \rightarrow 217.3\%$

$0.000098 \rightarrow 0.0098\%$

More On Percents

- General formula: 100*A/B = %
  - What percent of 4 is 3? 100*3/4 = 75%
  - What percent of 6 is 1? 100*1/6 ~ 16.667%
  - What percent of 7 is 1? 100*1/7 ~ 14.286%
- Pitfalls of Percentages
  - Know what the numerator is
  - Know what the denominator is

Calculating with Percentages

- Convert the percentage to a decimal
- Perform whatever calculation is needed in decimal arithmetic

Simple Interest

- Define P as the total amount of money, either in savings or borrowed
- Define r as the rate of interest per unit of time. Generally that unit is a month, or a year. Convert r into a decimal fraction.
- Define n as the number of units of time that have passed
- The amount of interest on P dollars at interest rate r held for n units of time is P*r*n
- $1000 held for 16 months at 0.4% interest per month yields $64
- $5000 held for 15 years at 5% interest per year yields $3750

Compound Interest

- Define P, r and n as with simple interest
- Define A as the total amount at the end of n^{th} unit of time
- $A = P*(1+r)^n$
- $5000 at 5% per year, compounded monthly, held for 15 years
  - P = 5000, r = 0.05/12 ~ 0.0041667, n = 12*15 = 180
  - $A = \$5000*(1.0041667)^{180} = \$10,568.52$ (rounded off to the nearest penny)
- $50,000 at 4.8% per year, compounded monthly, held for 25 years
  - P=50,000; r = 0.048/12 = 0.004; n = 12*25 = 300
  - $A=\$50,000*(1.004)^{300} = \$165,608.97$

Loans and Saving

- Set P = the payment
- Set A = the amount in question
- Set n = number of units of time
- Set r = interest per unit of time

Loans and Savings

Formula for payment

$$P = A*r/((1+r)^n -1)*(1+r))$$

Saving and Interest

- Say you need to have $8,000 in 7 years and you can get 4.8% interest per year, compounded monthly.
  - A = 8000
  - r = 0.048/12 = 0.004
  - n = 7*12 = 84
  - P = A*r/((1+r)^{n} -1)*(1+r)) = $8000*0.004/((1.004^{84} -1)(1.004))
    = 8000*0.004/((1.004^{84} – 1)*(1.004)) = 32/(1.39840148331-1(*1.004) ~ 80
- Assume the interest rate is actually 3.6% compounded monthly
  - r = 0.036/12 = 0.003
  - A ~ $83.63

Loans and Saving

You need to borrow A dollars, paying it in monthly installments, at interest rate r per unit time, paid over n payments. What will the payment be?

$$P = A*r/(1-(1+r)^{-n})$$

You can afford a payment of P dollars a month. The interest rate is r per unit time, and you plan to pay this over n units. What can you afford to borrow?

$$A= P*(1-(1+r)^{-n})/r$$

Payment on a Loan

- Borrowing $95,000 at 6% per annum, compounded monthly, paying back over 20 years
  - A = $95000
  - r = 0.06/12 = 0.005
  - n = 20*12 = 240 months
  - $P=95000*0.005/((1-(1.005)^{-240}) \sim \$680.61$
  - Total of payments: $163,336.40
- The same scenario, but at 5.4% interest
  - r = 0.054/12 = 0.0045
  - $P = 95000*0.0045/((1-(1.0045)^{-240}) \sim \$648.14$
  - Total of 240 payments: $155,553.60

What Can I Afford to Borrow?

- You can afford a payment of $700 a month, at 6% interest
  - P = 700
  - r = 0.06/12 = 0.005
  - n = 12*20 = 240
  - A = $700*(1-(1.005)^{-240})/0.005 = \$97,706.54$
- You find an interest rate of 5.4%
  - r = 0.054/12 = 0.0045
  - A = $700*(1-(1.0045)^{-240})/0.0045 = \$102,601.40$